RAYLEIGH FLOW – NUMERICALS ①

Points to remember

1) $h_{01} \neq h_{02} \Rightarrow T_{01} \neq T_{02}$

2) $P_{01} \neq P_{02}$

3) p^*, T^*, T_0^* are used to relate between ① & ②

4) To find p_0, T_0 at a point, use isentropic table at that point

5) $p = \rho R T$, $M_1 = \frac{c_1}{\sqrt{\gamma R T_1}}$, $M_2 = \frac{c_2}{\sqrt{\gamma R T_2}}$

6) Mention of combustion chamber / indication of change in stagnation temp or stagnation enthalpy like increase/decrease in stagnation enthalpy, heat added / heat removed, value of $\dot{m}_f$, Q_f etc... indicates Rayleigh flow

7) We need Mach number at a point to get properties at that point so finding M is very important

8) Gas tables & scientific calculator are of great importance.

Q) Conditions of gas in a combustor entry are $p_1 = 0.348$ bar, $T_1 = 810$K, $c_1 = 60$ m/s. Determine Mach number, pressure, temperature and velocity at the exit if increase in stagnation enthalpy of gas between entry & exit is 1172.5 kJ/kg. Assume $C_p = 1.005$ kJ/kgK, $\gamma = 1.4$. Also find maximum increase in stagnation enthalpy possible for given inlet.

Sol: Given

$p_1 = 0.348$ bar

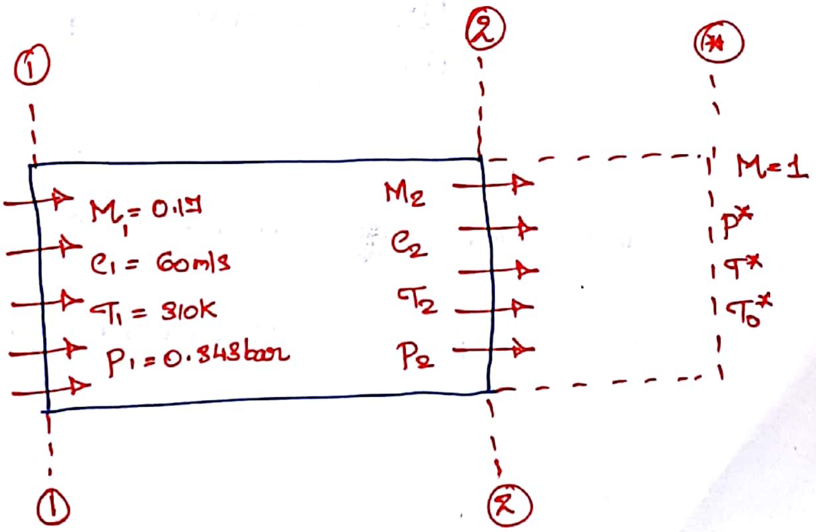
$T_1 = 810$ K

$c_1 = 60$ m/s

$h_{02} - h_{01} = 1172.5$ kJ/kg

$\gamma = 1.4$

$C_p = 1.005$ kJ/kgK



Mach number at inlet $M_1 = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{60}{\sqrt{1.4 \times 287 \times 310}} = 0.17$ ②

For $\gamma = 1.4$, $M_1 = 0.17$ from Rayleigh tables

M_1	P_1/p^*	P_{01}/P_0^*	$\frac{T_{01}}{T_0^*}$	$\frac{T_1}{T^*}$	$\frac{C}{C^*}$
0.17	2.306	1.2435	0.129	0.154	0.0665

$$P_1/p^* = 2.806 \Rightarrow p^* = \frac{P_1}{2.806} = \frac{0.843}{2.806} = 0.148 \text{ bar}$$

$$T_1/T^* = 0.154 \Rightarrow T^* = \frac{T_1}{0.154} = \frac{310}{0.154} = 2012.99 \text{ K}$$

$$h_{02} - h_{01} = C_p\,C\,T_{02} - T_{01}) = 1172.5 \text{ kJ/kg}$$

$$T_{02} - T_{01} = \frac{1172.5}{C_p} = \frac{1172.5}{1.005} = 1166.67 \text{ K}$$

From isentropic tables for $\gamma = 1.4$, $M = 0.17$

M_1	$\frac{T_1}{T_{01}}$	$\frac{P_1}{P_{01}}$
0.17	0.9948	0.980

$$\frac{T_1}{T_{01}} = 0.9943 \Rightarrow T_{01} = \frac{T_1}{0.9948} = \frac{310}{0.9943} = 311.78 \text{ K}$$

$$\frac{P_1}{P_{01}} = 0.980 \Rightarrow P_{01} = \frac{P_1}{0.980} = \frac{0.343}{0.980} = 0.35 \text{ bar}$$

$$T_{02} - T_{01} = 1166.67 \text{ K}$$

$$T_{02} = 1166.67 + T_{01} = 1166.67 + 311.78 = 1478.44 \text{ K}$$

on Rayleigh tables we had.

$$\frac{T_{01}}{T_0^*} = 0.129 \Rightarrow T_0^* = \frac{T_{01}}{0.129} = \frac{311.78}{0.129} = 2416.89\ K.$$

$$\frac{P_{01}}{P_0^*} = 1.2485 \Rightarrow P_0^* = \frac{P_{01}}{1.2485} = \frac{0.35}{1.2485} = 0.281\ bar$$

For exit condition we know $T_{02} = 1478.44\ K$ & $T_0^* = 2416.89K$

$$\left(\frac{T_0}{T_0^*}\right)_2 = \frac{1478.44}{2416.89} = 0.612$$

For $\gamma = 1.4$ $\frac{T_0}{T_0^*} = 0.612$ from Rayleigh tables.

M	P/P^*	P_0/P_0^*	T/T^*	T_0/T_0^*
0.44	1.888	1.189	0.690	0.597
0.449≈0.45.	1.870	1.185	0.708	0.612
0.46	1.852	1.181	0.725	0.680

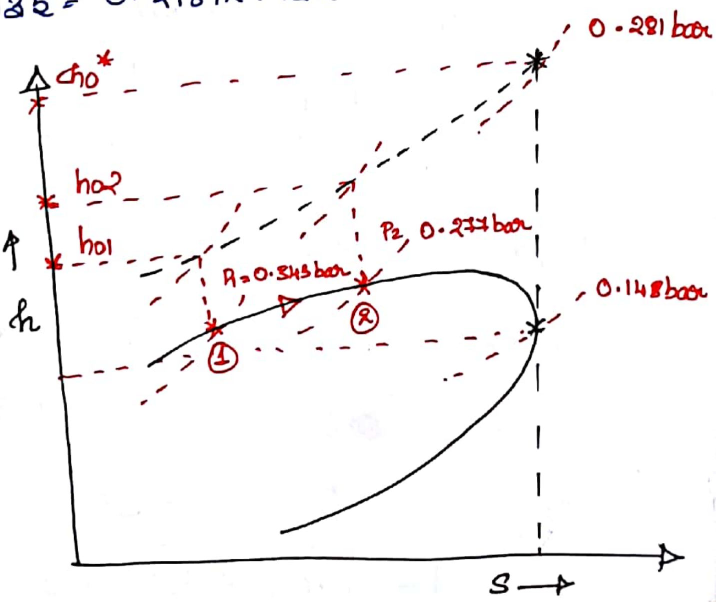
$$\frac{M-0.44}{0.46-0.44} = \underbrace{\frac{0.612-0.597}{0.68-0.62}}_{0.4545}$$

$$M_2 = 0.45.$$

$$\frac{P_2}{P^*} = 1.87 \Rightarrow P_2 = 1.87 \times P^* = 1.87 \times 0.148 = 0.277\ bar$$

$$\frac{T_2}{T^*} = 0.708 \Rightarrow T_2 = 0.708 \times T^* = 0.708 \times 2012.99 = 1425.2\ K$$

$$\frac{P_{02}}{P_0^*} = 1.185 \Rightarrow P_{02} = P_0^* \times 1.185 = 0.2181 \times 1.185 = 0.247\ bar$$



$$\frac{T_{02}}{T_0^*} = 0.612.$$

Maximum increase in stagnation enthalpy

$$= h_0^* - h_{01}$$

$$= c_p(T_0^* - T_{01})$$

$$= 1.005(2416.89 - 311.78)$$

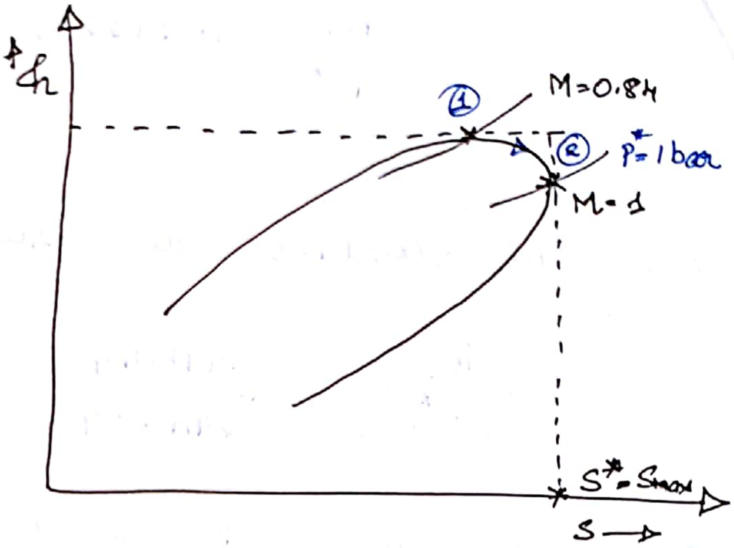
$$= 2115.64\ kJ/kg$$

8) If static condition of air at sonic state in ④ Rayleigh flow heat process are 1 bar and 500K, find pressure temperature and velocity of the maximum enthalpy point. What is change in entropy between these po

<u>Sol:</u>

Given

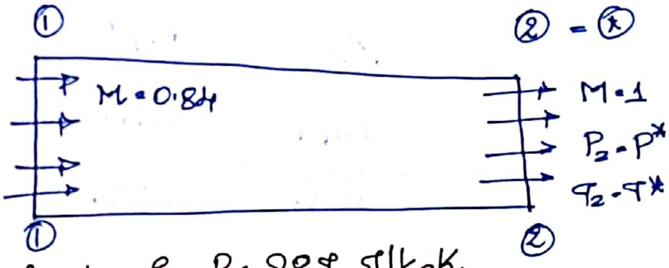
sonic state
$$\left\{\begin{array}{l} p^* = 1\ \text{bar} \\ T^* = 500\text{K} \end{array}\right.$$



In a Rayleigh heating process heat addition causes flow to accelerate in subsonic region & decelerate in supersonic region.

Its given in the question that the other point is maximum enthalpy point

$$\boxed{\text{In Rayleigh flow at maximum enthalpy point} \quad M = \frac{1}{\sqrt{\gamma}}}$$



Since there nothing is specified we take $\gamma = 1.4$ & $R = 287$ J/kgK.

Therefore Mach number at other point is $M = \frac{1}{\sqrt{1.4}} = 0.845 \approx 0.84$

So flow occurs between "M=0.84" & "M=1" & is a heating process.

Inlet condition is M = 0.84

Exit condition is M = 1.00

For $\gamma = 1.4$ M = 0.84 from Rayleigh flow tables

M_1	P_1/p^*	T_1/T^*	$\frac{c}{c^*}$
0.84	1.207	1.028	0.852

$P_1/p^* = 1.207 \Rightarrow P_1 = 1.207 \times p^* = 1.207 \times 1\ \text{bar} = \boxed{1.207\ \text{bar}}$

$T_1/T^* = 1.028 \Rightarrow T_1 = 1.028 \times T^* = 1.028 \times 500\text{K} = \boxed{514\ \text{K}}$

$c/c^* = 0.852 \Rightarrow c = 0.852 \times c^* = 0.852 \times \sqrt{1.4 \times 287 \times 500} = \boxed{381.88\ \text{m/s}}$

$$s_2 - s_1 = s^* - s_1 = -(s_1 - s^*) = -\frac{(s_1 - s^*)}{c_p} c_p = c_p \left[\ln \left[M^2 \left[\frac{1+\gamma}{1+\gamma M^2} \right] \right]^{\frac{\gamma+1}{\gamma}} \right]$$

$$\boxed{= 0.0258\ \text{kJ/kgK}}$$

velocity
these pa

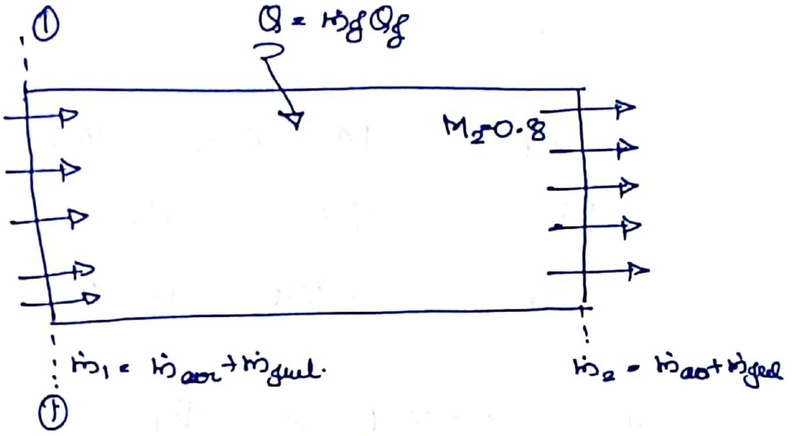
ir-fuel mixture enters combustion chamber with ⑤ 150m/s, 4 bar & 410K. The Mach number at exit of c.c. is 0.8. Take $\gamma = 1.\overset{3}{3}{}^{4}$ $C_p = 1.144 \text{ kJ/kgK}$ $Q_f = 48 \text{ MJ/kg}$

Find a) Entry Mach number 2) Exit velocity Temperature & pressure c) Stagnation pressure loss d) Air-fuel ratio required.

Sol:

Assuming the the air-fuel mixture is homogeneous and its a perfect gas

Given:



$C_1 = 150 \text{ m/s}$

$P_1 = 4 \text{ bar}$

$T_1 = 410 \text{ K}$

$M_2 = 0.8$

$\gamma = 1.3$

$C_p = 1.144 \text{ kJ/kgK}$

$Q_f = 48 \times 10^6 \text{ J/kg}$

$$\frac{C_p}{C_v} = \gamma$$

$$C_p - C_v = R \Rightarrow C_p - \frac{C_p}{\gamma} = R = C_p(1 - 1/1.3) = 264 \text{ J/kgK}$$

Inlet Mach Number $M_1 = \dfrac{C_1}{\sqrt{\gamma R T_1}} = \dfrac{150}{\sqrt{1.3 \times 264 \times 410}} = 0.3999 \approx \boxed{0.4}$

For $\gamma = 1.4$ from Rayleigh tables

M	P/P^*	P_0/P_0^*	T/T^*	T_0/T_0^*	C/C^*
0.4	1.961	1.157	0.615	0.529	0.814
0.8	1.266	1.019	1.025	0.964	0.816

$$\frac{P_1}{P^*} = 1.157 \Rightarrow P^* = \frac{P_1}{1.157} = \frac{4 \text{ bar}}{1.157} = 3.457 \text{ bar}$$

$$\frac{T_1}{T^*} = 0.615 \Rightarrow T^* = \frac{T_1}{0.615} = \frac{410}{0.615} = 666.67 \text{ K}$$

$\frac{c_1}{c^*} = 0.814 \Rightarrow c^* = \frac{c_1}{0.814} = \frac{150}{0.814} = 477.71$ m/s

$\frac{T_2}{T^*} = 1.025 \Rightarrow T_2 = 1.025 \times T^* = 1.025 \times 666.67 = \boxed{688.34}$

$\frac{P_2}{P^*} = 1.266 \Rightarrow P_2 = 1.266 \times P^* = 1.266 \times 3.457 = \boxed{4.376 \text{ bar}}$

$\frac{C_2}{C^*} = 0.810 \Rightarrow C_2 = 0.81 \times C^* = 0.81 \times 477.71 = \boxed{386.945 \text{ m/s}}$

For $\gamma = 1.4$, $M = 0.4$ from isentropic tables.

M_1	T_1/T_{01}	P_1/P_{01}
0.4	0.969	0.895

$\frac{T_1}{T_{01}} = 0.969 \Rightarrow T_{01} = \frac{T_1}{0.969} = \frac{410}{0.969} = 423.12$ K

$\frac{P_1}{P_{01}} = 0.895 \Rightarrow P_{01} = \frac{P_1}{0.895} = \frac{4}{0.895} = 4.469$ bar

For $\gamma = \overset{1.3}{\cancel{1.4}}$, $M = 0.4$ from Rayleigh tables we have.

$\frac{P_{01}}{P_0^*} = 1.157 \Rightarrow P_0^* = \frac{P_{01}}{1.157} = \frac{4.469}{1.157} = 3.863$ bar

For $\gamma = \overset{1.3}{\cancel{1.4}}$ $M = 0.8$ from Rayleigh tables, we have.

$\frac{P_{02}}{P_0^*} = 1.019 \Rightarrow P_{02} = 1.019 \times P_0^* = 1.019 \times 3.863 = 3.936$ bar

$\Delta P_0 = P_{01} - P_{02} = 4.469 - 3.936 = \boxed{0.533 \text{ bar}}$

heat add $\dot{Q} = C_p(T_{02} - T_{01})$

Ans Gas tables (Rayleigh flow) for $\gamma = \overset{1.3}{\cancel{1.4}}$ ⑦ $M = 0.4$

$$\frac{T_{01}}{T_0^*} = 0.529 \Rightarrow T_0^* = \frac{T_{01}}{0.529} = \frac{428.12}{0.529} = 799.85\,K.$$

For $\gamma = \overset{1.3}{\cancel{1.4}}$, $M = 0.8$ from Rayleigh flow tables

$$\frac{T_{02}}{T_0^*} = 0.964 \Rightarrow T_{02} = 0.964 \times T_0^* = 0.964 \times 799.85$$

$$T_{02} = 771.0554\,K.$$

$$\dot{Q} = 1.144 \times (771.055 - 428.12) = 398.038\ kJ/kg$$

Inlet mass flow rate $\dot{m} = \rho_1 A C_1 = \frac{P_1}{RT_1} \times A \times C_1.$

$$\dot{m}_1 = \frac{(4 \times 10^5)\ N/m^2}{264 \times 410} \times 1 \times 150 = 554.324\ kg/s.$$

$$\dot{m}_1 = \dot{m}_{air} + \dot{m}_{fuel}.$$

Total heat added / time $= \dot{m}_f Q_f$

Total heat added / time $= \dot{Q} \times \dot{m}_1$ { $\dot{Q} \rightarrow$ in kJ/kg to get total heat added multiply by kg/s (power) }

$$\dot{m}_f Q_f = \dot{Q} \times \dot{m}_1.$$

$$\dot{m}_f = \frac{\dot{Q} \times \dot{m}_1}{Q_f} = \frac{398.038 \times 554.324}{43000} = 5.131\ kg/s$$

$$\dot{m}_{air} = \dot{m}_1 - \dot{m}_{fuel} = 554.324 - 5.131 = 549.193\ kg/s.$$

$$\text{Air-fuel ratio} = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} = \frac{549.193}{5.131} = \boxed{107.034 : 1}$$

8) A gas at pressure of 0.7 bar & 280K enter a ⑧ C.C. at veloc
Heat supplied in C.C. is 1500 kJ/kg. Determine Mach num
pressure and Temperature at exit.

Sol:

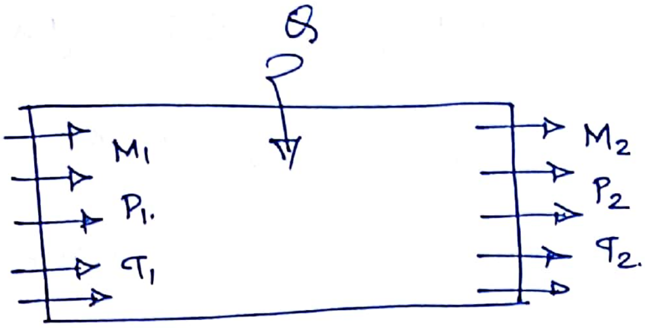
Given

$P_1 = 0.7$ bar

$T_1 = 280$ K

$C_1 = 55$ m/s

$\dot{Q} = 1500$ kJ/kg



$$M_1 = \frac{C_1}{\sqrt{\gamma R T_1}} = \frac{55}{\sqrt{1.4 \times 287 \times 280}} = 0.164$$

For $\gamma = 1.4$, $M_1 = 0.164$ from Rayleigh tables

M_1	P_1/P^*	T_1/T^*	T_{01}/T_0^*
0.16	2.317	0.137	0.115

$$\frac{P_1}{P^*} = 2.317 \Rightarrow P^* = \frac{P_1}{2.317} = \frac{0.7}{2.317} = 0.302 \text{ bar}$$

$$\frac{T_1}{T^*} = 0.137 \Rightarrow T^* = \frac{T_1}{0.137} = \frac{280}{0.137} = 2043.8 \text{ K}$$

From isentropic ~~M2~~ tables $\gamma = 1.4$, $M = 0.164$ $\frac{T_1}{T_{01}} = 0.9949$

$$T_{01} = T_1/0.9947 = 280/0.9947 = 281.491 \text{ K}$$

heat added $Q = h_{02} - h_{01} = C_p(T_{02} - T_{01})$

$$T_{02} - T_{01} = \frac{Q}{C_p} = \frac{1500}{1.005} = 1492.537 \text{ K}$$

$$T_{02} = 1492.537 + T_{01} = 1492.537 + 281.49 = 1774.03\text{K}$$

From Rayleigh tables for $\gamma = 1.4$ $M = 0.164$ $\frac{T_{01}}{T_0^*} = \overset{0.115}{\cancel{0.202}}$

$$T_0^* = \frac{T_{01}}{0.115} = \frac{281.49}{0.115} = 2447.74 \text{ K}$$

know at exit $T_{02} = 1774.08 K$ & $T_0^* = 2447.74 K.$

$$\frac{T_{02}}{T_0^*} = \left(\frac{T_0}{T_0^*}\right)_2 = \frac{1774.08}{2447.74} = 0.725.$$

for $\gamma = 1.4$, $\left(\frac{T_{02}}{T_0^*}\right) = 0.725$ from Rayleigh tables. $\boxed{M = 0.52}$

M_2	P_2/p^*	T_2/T^*	T_{02}/T_0^*	c/c^*
0.52	1.774	0.819	0.725	0.471

$$\frac{P_2}{p^*} = 1.741 \Rightarrow P_2 = 1.741 \times p^* = 1.741 \times 0.302 = \boxed{0.526 \text{ bar}}$$

$$\frac{T_2}{T^*} = 0.819 \Rightarrow T_2 = 0.819 \times T^* = 0.819 \times 2043.8 = \boxed{1678.87 K.}$$